MATH 534 – HOMEWORK 3

(1) Let G be a cyclic group that has *exactly* three subgroups: G itself, the trivial subgroup, and a proper subgroup of order 7. Find $|G|$, making sure to justify your answer.

(2) Let G be an abelian group with $|G| = 35$, and suppose that every element $g \in G$ satisfies the equality $g^{35} = e$. Prove that G is cyclic. (hint: I claim the result follows once you have your hands on an element of order 5 and an element of order 7. Think of the possible of orders of elements in the group, and deduce that you must have elements as above. Finally, at some point you might use the fact that neither 4 nor 6 divide 34.)

(3) Let G be a group.
  (a) Let $H \leq G$ and $K \leq G$ be subgroups. Show that $H \cap K \subseteq G$ is a subgroup of G.

  (b) Let $a, b \in G$ such that $|a|$ and $|b|$ are finite and relatively prime. Show that $\langle a \rangle \cap \langle b \rangle = \{e\}$.

(4) Let $G = \mathbb{Z}/30$.
  (a) How many (distinct) subgroups does G have?

  (b) Draw the lattice of subgroups for G.