# MATH 534 – HOMEWORK 5

(1) Let $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 4 & 6 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 2 & 4 & 3 & 5 \end{pmatrix}$.

(a) Compute the following: $\alpha^{-1}$, $\beta^{-1}$, $\alpha\beta$, and $\beta\alpha$.

(b) Write $\alpha$ and $\beta$ as products of disjoint cycles.

(c) Compute $\beta^{-1}\alpha\beta$ and $\alpha^{-1}\beta\alpha$ and write them as products of disjoint cycles. How do their "cycle structures" compare to those of $\alpha$ and $\beta$ (respectively)?

(2) How many elements of order 4 are there in $\mathfrak{S}_6$? How many of order 2? Justify your answers.

(3) A *perfect shuffle* is performed on a deck of cards by splitting the deck into two halves (the top and the bottom half, we're assuming our deck has an even number of cards), then interweaving the two halves so that every-other card comes from the same half (and the original order within the halves is preserved). There are two ways to do this: an *out shuffle* preserves the first and last cards, while an *in shuffle* does not.

(a) Show that after 8 perfect **out** shuffles, a deck of 52 cards is returned to its original position, but no fewer number of perfect out shuffles will do this.

(b) How many perfect **in** shuffles does it take to return a deck of 10 cards to its original position?

(4) Let $(ab)$ and $(cd)$ be distinct 2-cycles in $\mathfrak{S}_n$. Show that these elements commute if and only if they are disjoint. Using this, show that $\mathfrak{S}_n$ is not abelian if $n \geq 3$.

(5) Let $\alpha = (123)(145) \in S_5$. Compute $\alpha^{99}$.

(6) Show that we cannot find an element $\sigma \in S_7$ so that $\sigma^2 = (1234)$.
*Hint: what would be the possible orders for such a $\sigma$?*