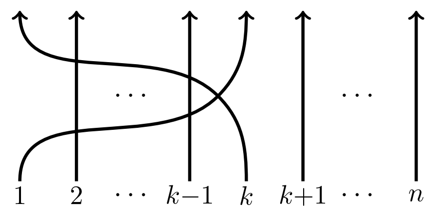
MATH 534 – HOMEWORK 6

(1) Prove that every element in S_n (for $n > 1$) can be written as a product of transpositions of the form $(1\ k)$.
Hint: a quick proof uses string diagrams (there are others). By our results from class, it suffices to show that $(i\ \ i{+}1)$ *can be written as product of transpositions of the above form. You could try to induct on* i *(think about the* $i = 2$ *case to get the general idea), and use the fact that one string diagram for* $(1\ k)$ *is given by:*



(2) Prove that the 4-cycle $(1234) \in S_n$ cannot be written as the product of 3-cycles.

(3) Show that a permutation of odd order must be an even permutation. Is every permutation of even order an odd permutation?

(4) Prove that the groups $(\mathbb{Z}, +)$ and $(\mathbb{Q}, +)$ are not isomorphic. (*Hint: try to describe a homomorphism* $\phi : \mathbb{Z} \to \mathbb{Q}$ *in terms of* $\phi(1)$)

(5) Let $\phi : G_1 \to G_2$ be a homomorphism.
  (a) Define the *image* of G_1 under ϕ to be:
$$\phi(G_1) = \{g \in G_2 \mid g = \phi(a) \text{ for some } a \in G_1\}$$
  Prove that this is a subgroup of G_2.

  (b) Define the *kernel* of ϕ to be:
$$\ker \phi = \{a \in G_1 \mid \phi(a) = e\}$$
  where here $e \in G_2$ is the identity. Prove that this is a subgroup of G_1.

(6) Is the function $\varphi : \mathbb{Z}/12 \to \mathbb{Z}/10$ which is given by $\varphi(k) = 3k$ (where the latter is interpreted as an element in $\mathbb{Z}/10$) a homomorphism? If so, prove it and if not, explain why not.