

MATH 534 – HOMEWORK 7

- (1) An isomorphism from a group to itself, *i.e.* an isomorphism $\alpha : G \rightarrow G$, is called an automorphism. Suppose that G is a finite abelian group which has no elements of order 2. Show that the function $\alpha(g) = g^2$ is an automorphism of G . Show by example that the result doesn't hold in the case that G is infinite.
- (2) Consider the group $G = (\mathbb{Z}, +)$.
 - (a) Show that G is isomorphic to the proper subgroup $H = \{2k \mid k \in \mathbb{Z}\}$ of even elements.
 - (b) Show that there are in fact infinitely many subgroups of G to which it is isomorphic.
- (3) Consider the group $\mathbb{R}^\times = (\mathbb{R} \setminus \{0\}, \times)$ of non-zero real numbers under multiplication. Prove that this group is **not** isomorphic to $(\mathbb{R}, +)$.
- (4) Explain why \mathfrak{S}_8 contains subgroups isomorphic to $\mathbb{Z}/15$, $(\mathbb{Z}/16)^\times$, and D_8 . Here, D_8 denotes the group of symmetries of a regular convex octagon.