MATH 534 – HOMEWORK 8

(1) Let $H, K \leq G$ be two subgroups of a given group G. Show that for $a \in G$ the coset $a(H \cap K)$ is equal to $(aH) \cap (aK)$, i.e. equal to the intersection of the cosets aH and aK.

(2) Use the Theorem of Lagrange (and its consequences) to show that $|(\mathbb{Z}/n)^{\times}|$ is always even when $n > 2$.

(3) Suppose G is a finite abelian group and $|G|$ is odd. Show that the product of all of the elements in G is the identity. Is the same true if $|G|$ is even?

(4) Let $|G| = 8$. Prove that G has an element of order 2.

(5) Let G be a group and let $H, K \leq G$ be subgroups satisfying $|H| = 20$ and $|K| = 28$. Prove that $H \cap K$ is abelian. (Hint: Start by computing the order of $H \cap K$. We've seen that $H \cap K$ is a subgroup of G, but it can also be viewed as a subgroup of...)