# MATH 534 – HOMEWORK 9

(1) Let $N \trianglelefteq G$ be a normal subgroup. Finish our proof that $G/N$ is indeed a group by showing that the group axioms I, II, and III hold (i.e. show the binary operation we defined is associative, and show that $G/N$ has an identity element and inverses).

(2) Find a direct product of cyclic groups to which $(\mathbb{Z}/24)^\times$ is isomorphic.

(3) Characterize the positive integers $n$ for which every abelian group of order $n$ is cyclic.

(4) Let $(R, +, \cdot)$ be a ring with identity $1 \in R$. Prove the following:

  (a) The identity 1 is unique

  (b) If $a \in R^\times$, then its inverse is unique

  (c) $(R^\times, \cdot)$ is a group

(5) Let $(R, +, \cdot)$ be a ring such that $(R, +)$ is a cyclic group. Prove that $R$ is a commutative ring.